PUBLICATIONS OF THE ASTRONOMICAL SOCIETY OF THE PACIFIC
91:577–584, August 1979

ON THE PROBABILITY OF CLEAR DAYTIME SKIES NEAR KITT PEAK NATIONAL OBSERVATORY

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Received 1979 February 20

Graphs are given for the probability of getting a required number of clear days at KPNO during an observing period of given length. The probabilities undergo large variations during the year, and are dominated by the persistence of weather patterns. The analysis is based on records from the Tucson International Airport.

Key words: Kitt Peak—cloud cover—observing conditions

A substantial part of the solar observing time at Kitt Peak National Observatory is allocated to short-term visitors who may be present for only a few days. If the skies are cloudy, their observing sessions are doomed. Therefore the probability of good skies is of more than usual interest. No specific records of cloud cover are kept for Kitt Peak itself, but a good series is available from Tucson International Airport, some 40 miles away. The same weather pattern usually envelopes the two sites, though the Observatory is sometimes more affected by cloud because of its greater altitude.

The cloud records are given in the "Local Climatological Data" for Tucson, published by the U.S. Department of Commerce's agency ESSA, later NOAA. These include the mean daily cloud cover, sunrise to sunset. For present purposes, two levels have been considered, (i) zero cloud and (ii) mean cloud not exceeding 0.2 of the sky. The latter seems to be a useful guide to good skies at Kitt Peak, bearing in mind the poorer conditions sometimes found there (on the one hand, many programs can be carried out with mean cloud in excess of 0.2 of the sky; on the other, cloud or fog may form over the mountain but scarcely affect the surrounding regions including Tucson).

Table I lists for each month the mean number of days falling into the above classes during 1968–77. For comparison, the average daily cloud cover (sunrise to sunset) is listed in Table II. Big annual variations occur, conditions being best in May–June and September–October, very poor in July–August, and again rather poor in midwinter.

Special attention has been given to the duration of sunny periods, defined as those in which the mean cloud cover, sunrise to sunset, remains within class (i) or class (ii) on successive days. For these purposes the shortest sunny period is one day. Table III lists the total numbers of sunny periods of given duration *commencing* in given months over the ten-year period (except for December, for which data have been available for only nine years).

We can test these data readily to establish whether the probability of a sunny day being followed by a sunny day is random. If so, the probability being β, then in any sunny period the probability of n successive sunny days is β^{n-1}, and the mean duration of a sunny spell is

$$d = (1-\beta) \sum_{n=1}^{\infty} n\,\beta^{n-1} = \frac{1}{1-\beta} \quad . \qquad (1)$$

And of N sunny periods, the number N_n having durations of at least n days is $N\beta^{n-1}$, so that

$$\log N_n = \log N + (n-1)\log\beta \quad , \qquad (2)$$

i.e., the plot of $\log N_n$ against n is linear. We may find the N_n by summing along the rows of Table III, so that equation (2) may be tested against observations. Figures 1(a) and 1(b) show the results. For cloudless days (case (i)) the plots are clearly linear in six of the months, they seem to be convex upward for two, concave upward for three, while there are too few sunny periods in July for proper analysis. In case (ii), eleven of the months show clearly linear plots, only one (May) showing an apparent nonlinearity. But the numbers of points are small where any of the curves show apparent nonlinearities, and the fluctuations are large. In any case, there is no systematic departure from linearity, and the evidence is overwhelming that β is effectively independent of n; the probability of a sunny day being followed by a sunny day is independent of the preceding number of sunny days.

A second test verifies this result. From equation (1) the duration of a sunny period depends only on β. Table IV gives separately for cases (i) and (ii) the mean observed durations as derived from Table III. But nearly one-half of the sunny periods last for only one day. We can make a second calculation which will

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TABLE I

Monthly Average Number of Days of Good Sky 1968-77

Class	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
(i)	9.6	7.1	9.3	13.2	14.6	15.9	1.8	4.0	10.2	14.8	11.6	11.1
(ii)	13.3	12.1	14.4	18.3	20.4	21.2	6.3	10.6	17.3	19.0	16.1	14.4

Note: Class (i) is cloudless.
Class (ii) consists of days whose mean cloud cover, sunrise to sunset, does not exceed 0.2.

TABLE II

Daily Cloud Cover* 1968-77

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
0.43	0.43	0.39	0.28	0.24	0.19	0.53	0.40	0.30	0.27	0.34	0.44

*Mean fraction of sky covered by cloud, sunrise to sunset.

TABLE III

Total Numbers of Sunny Periods 1968-77

Class (i)

	Duration (days)											Mean Periods per Month	Mean Duration (days)
Month	1	2	3	4	5	6	7	8	9	10	>10		
Jan	18	10	8	4	2							4.2	2.10
Feb	28	12	2	1	1							4.4	1.52
Mar	36	8	10	0	1	1						5.6	1.66
Apr	17	12	6	3	4	4	0	0	1	1		4.8	2.79
May	18	14	6	6	6	1	1	2	0	0	1 of 11d.	5.5	2.87
Jun	16	7	8	3	1	2	0	2	1	1	1 of 12d. 1 of 13d.	4.3	3.33
Jul	8	1	0	0	0	1						1.0	1.60
Aug	14	4	3	1	0	1						2.3	1.78
Sep	14	8	4	4	3	1	0	0	2	0	1 of 12d.	3.7	2.95
Oct	25	17	6	3	5	1	0	0	2	1	1 of 11d.	6.1	2.61
Nov	26	11	5	0	3	0	1	0	0	0	2 of 12d.	4.8	2.27
Dec*	17	9	6	6	1	1						4.4	2.20

*Data are available in December for 9 years only.

yield the mean duration if the probability β is independent of the preceding number of sunny days; for this the first column of data in Table III is deleted and the durations renumbered starting from the original second column. Table IV includes these results, which agree well with the first column except for July–August, where the numbers of sunny periods are small. The good agreement verifies the constancy of β.

TABLE III

Total Numbers of Sunny Periods 1968-77

Class (ii)

Month	Duration (days) 1	2	3	4	5	6	7	8	9	10	>10	Mean Periods per Month	Mean Duration (days)
Jan	22	9	8	4	4	2	0	1	1	1		5.2	2.67
Feb	25	13	13	2	1	1	1	0	0	0	1 of 11d.	5.7	2.23
Mar	23	13	8	2	3	3	3	1				5.6	2.55
Apr	14	5	8	4	4	4	0	5	0	1	1 of 11d. 2 of 12d.	4.8	4.02
May	13	3	8	4	7	6	4	4	0	0	1 of 11d. 1 of 12d.	5.1	4.18
Jun	4	7	4	3	8	2	2	0	0	0	1 of 12d. 3 of 14d. 1 of 16d. 1 of 20d.	3.6	5.50
Jul	17	2	3	3	0	0	1	1				2.7	2.11
Aug	25	1	6	4	3	3	1	0	1	0	1 of 11d.	4.5	2.69
Sep	17	5	5	8	5	0	0	2	2	2	1 of 12d.	4.7	3.51
Oct	12	16	9	2	3	7	0	1	1	0	2 of 11d. 1 of 12d. 1 of 16d.	5.5	3.69
Nov	19	13	6	3	1	0	2	1	1	0	1 of 11d.	4.7	2.60
Dec*	16	9	7	5	4	1	2					4.9	2.45

*Data are available in December for 9 years only.

The values of β may now be obtained from the mean durations of Table III, using equation (1). These are given in Table V (second column), together with the probability $\alpha = S/D$ that an arbitrary day will be sunny, S being the mean number of sunny days (Table I) per month of length D. The values of β are systematically and substantially greater than those of α; good skies are far more persistent than with purely random weather. The probability, γ, that a cloudy day is followed by a sunny day is given by $\gamma = N(D - S)$, where N is the mean number of sunny periods per month. (Table III, thirteenth column), and $D - S$ is the mean number of cloudy days per month.

The probability of a given number of sunny days occurring during an arbitrary observing period is an interesting statistical problem because of the inequalities of α, β, and γ due to the persistence of weather patterns. The probability of the first day in the observing period being sunny is α; the probability of at least one sunny day occurring in a period of d days is

$$\begin{aligned} &\alpha + (1-\alpha)\,\gamma + (1-\alpha)\,(1-\gamma)\,\gamma + (1-\alpha)\,(1-\gamma)^2\gamma + \ldots + (1-\alpha)(1-\gamma)^{d-2}\,\gamma \\ &= \alpha + (1-\alpha)\,\gamma \left\{ \frac{1-(1+\gamma)^{d-1}}{1-(1-\gamma)} \right\} \qquad (3) \\ &= 1 - (1-\alpha)\,(1-\gamma)^{d-1} \quad . \end{aligned}$$

The probability of at least two sunny days occurring in a period of d days is

$$\begin{aligned}
&\alpha[\beta + (1-\beta)\gamma + (1-\beta)(1-\gamma)\gamma + (1-\beta)(1-\gamma)^2\gamma + \cdots + (1-\beta)(1-\gamma)^{d-3}\gamma] \\
&+ (1-\alpha)\gamma[\beta + (1-\beta)\gamma + (1-\beta)(1-\gamma)\gamma + \cdots \qquad \cdots + (1-\beta)(1-\gamma)^{d-4}\gamma] \\
&+ (1-\alpha)(1-\gamma)\gamma[\beta + (1-\beta)\gamma + \cdots \qquad \cdots + (1-\beta)(1-\gamma)^{d-5}\gamma] \\
&+ \cdots \\
&= \alpha\left[\beta + (1-\beta)\gamma\left\{\frac{1-(1-\gamma)^{d-2}}{1-(1-\gamma)}\right\}\right] \\
&+ (1-\alpha)\gamma[\beta + (1-\beta)\{1-(1-\gamma)^{d-3}\}] \\
&+ (1-\alpha)(1-\gamma)\gamma[\beta + (1-\beta)\{1-(1-\gamma)^{d-4}\}] \\
&+ \cdots \\
&= \alpha\left[\beta + (1-\beta\left\{1-(1-\gamma)^{d-2}\right\}\right] + (1-\alpha)\gamma\sum_{n=0}^{d-3}(1-\gamma)^n\left[\beta + (1-\beta)\left\{1-(1-\gamma)^{d-3-n}\right\}\right] \\
&= \alpha\left[1-(1-\beta)(1-\gamma)^{d-2}\right] + (1-\alpha)\gamma\left[\sum_{n=0}^{d-3}(1-\gamma)^n - (1-\beta)\sum_{n=0}^{d-3}(1-\gamma)^{d-3}\right] \\
&= \alpha[1-(1-\beta)(1-\gamma)^{d-2}] + (1-\alpha)[1-(1-\gamma)^{d-2} - \gamma(1-\beta)(d-2)(1-\gamma)^{d-3}] \qquad (4)
\end{aligned}$$

TABLE IV

Mean Durations of Sunny Periods

	case (i) Obs.	case (i) Obs. without Table III 1st Col.	case (ii) Obs.	case (ii) Obs. without Table III 1st Col.
Jan	2.10	1.92	2.67	2.90
Feb	1.52	1.44	2.23	2.28
Mar	1.66	1.85	2.55	2.72
Apr	2.79	2.77	4.02	4.26
May	2.87	2.78	4.18	4.26
Jun	3.33	3.70	5.50	5.38
Jul	1.60	3.0	2.11	3.0
Aug	1.78	2.0	2.69	3.8
Sep	2.95	3.13	3.51	4.27
Oct	2.61	2.72	3.69	3.44
Nov	2.27	2.77	2.60	2.68
Dec	2.20	2.09	2.45	2.54

The probability that an arbitrary number n of sunny days occurs in d consecutive days is more easily obtained by using a recurrence relation. If the probability of having n sunny days out of d, the last day being sunny, is ${}^dF_n$; and that of having n sunny days out of d, the last day being cloudy, is ${}^dC_n$, then the probabili-

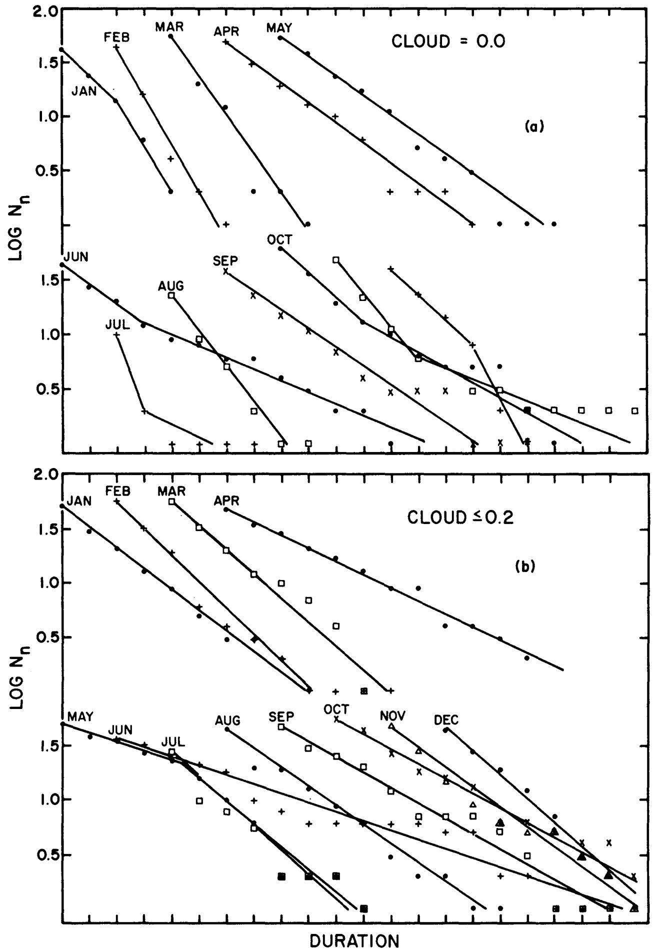

FIG. 1—Plots of $\log N_n$ against time, where N_n is the number of sunny periods having durations of at least n days (see eq. (2)); along the abscissa each mark indicates an interval of one day. The plots are for the total numbers of sunny periods in a given calendar month, integrated over ten years (nine years for December). In Figure 1(a) (above), the data are for days of zero cloud, sunrise to sunset; in Figure 1(b) (below), the mean cloud cover sunrise to sunset, does not exceed 0.2 of the sky. Except when the numbers are small, the plots are linear, demonstrating that β (eqs. (1), (2)) is constant for any month, irrespective of duration of a sunny period. Plots for successive months are displaced for clarity.

ty of $d + 1$ days having n sunny with the last sunny is

$$^{d+1}F_n = {}^dF_{n-1}\,\beta + {}^dC_{n-1}\,\gamma \quad ,$$

while that of n sunny with the last cloudy is

TABLE V

The Probabilities of Good Skies

	Case (i)			Case (ii)		
	α	β	γ	α	β	γ
Jan	0.31	0.52	0.20	0.43	0.63	0.29
Feb	0.25	0.34	0.21	0.43	0.55	0.35
Mar	0.30	0.40	0.26	0.46	0.61	0.34
Apr	0.44	0.64	0.29	0.61	0.75	0.41
May	0.47	0.65	0.34	0.66	0.76	0.48
Jun	0.53	0.70	0.30	0.71	0.82	0.41
Jul	0.06	0.37	0.03	0.20	0.53	0.11
Aug	0.13	0.44	0.09	0.34	0.63	0.22
Sep	0.34	0.66	0.19	0.58	0.72	0.37
Oct	0.48	0.62	0.38	0.61	0.73	0.50
Nov	0.39	0.56	0.26	0.54	0.62	0.34
Dec	0.36	0.55	0.22	0.46	0.59	0.29

$$^{d+1}C_n = {}^dF_n(1-\beta) + {}^dC_n(1-\gamma) \quad .$$

But for the first day of an arbitrary observing period,

$$^1F_1 \equiv 1 - {}^1C_0 \equiv \alpha \qquad {}^1F_1 \equiv 1 - {}^1C_0 \equiv \alpha$$

while

$$^dC_0 = (1-\alpha)(1-\gamma)^{d-1} \quad .$$

From these relations, the total probability, $^dF_n + {}^dD_n$ of having n sunny days out of d, can be calculated readily, and the probability of having at least n sunny days follows.

Figure 2 shows probabilities for each month. The values depend very much on the arbitrary probability that the first day will be sunny. Once the first day has occurred, the probabilities for the remainder of the session can be recast; they will be lowered substantially if the first day is cloudy, or raised correspondingly if the first day is sunny.

Good skies are most likely in June and May, and are very infrequent in July and August. However, sunny skies are not necessarily sufficient to ensure good observing conditions, for wind and poor seeing can affect programs adversely. Again, some types of observation may be very successful with a comparatively poor average cloud cover sunrise to sunset. Clearly the present analysis cannot deal in detail with the prospects of success for programs having widely differing requirements.

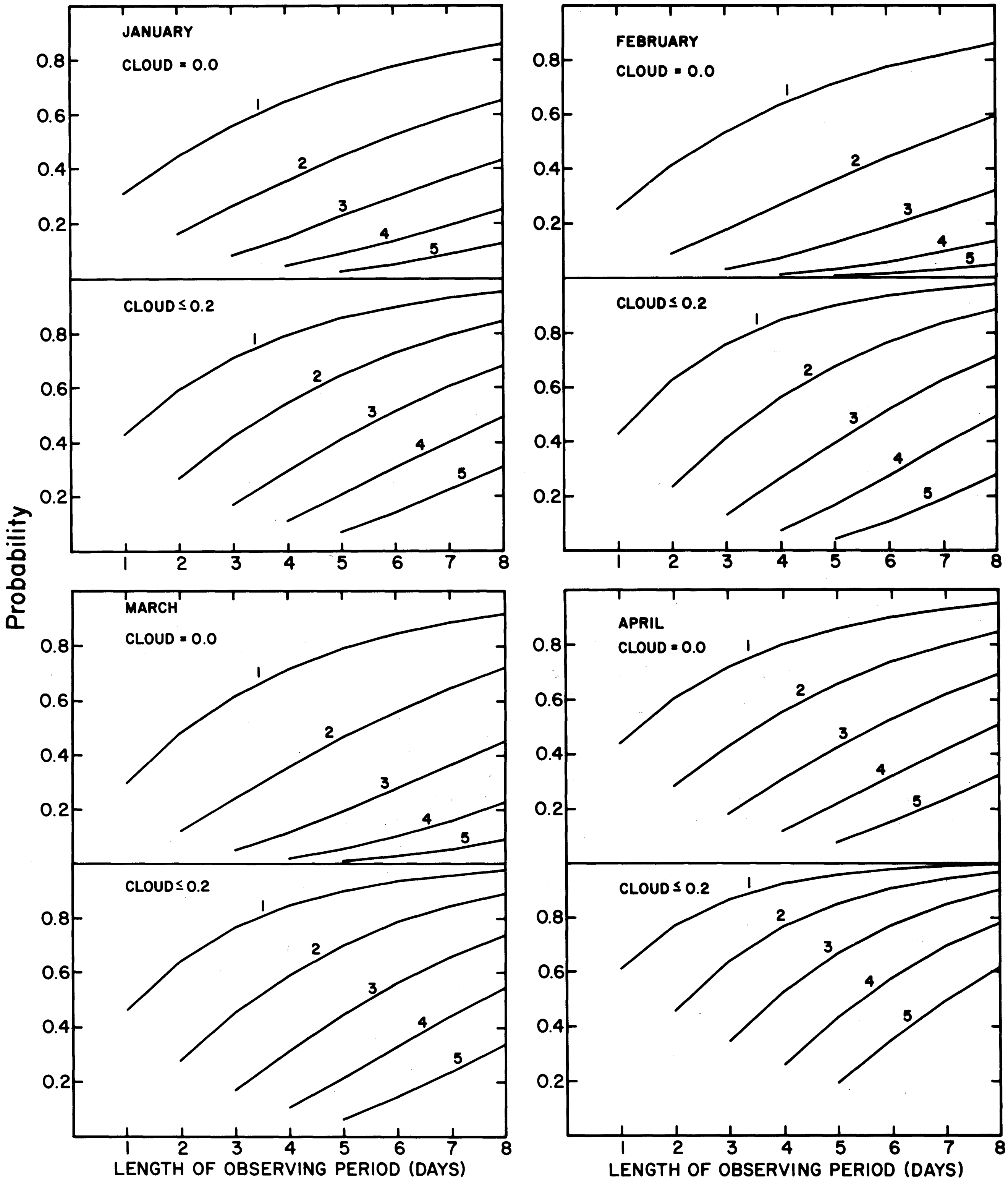

FIG. 2-1—The probability of obtaining not less than 1, 2, 3, 4, or 5 sunny days (indicated alongside the plots) during an arbitrary observing period of given length commencing in a given month. The probabilities vary greatly from month to month, being highest in June and May and lowest in July and August.

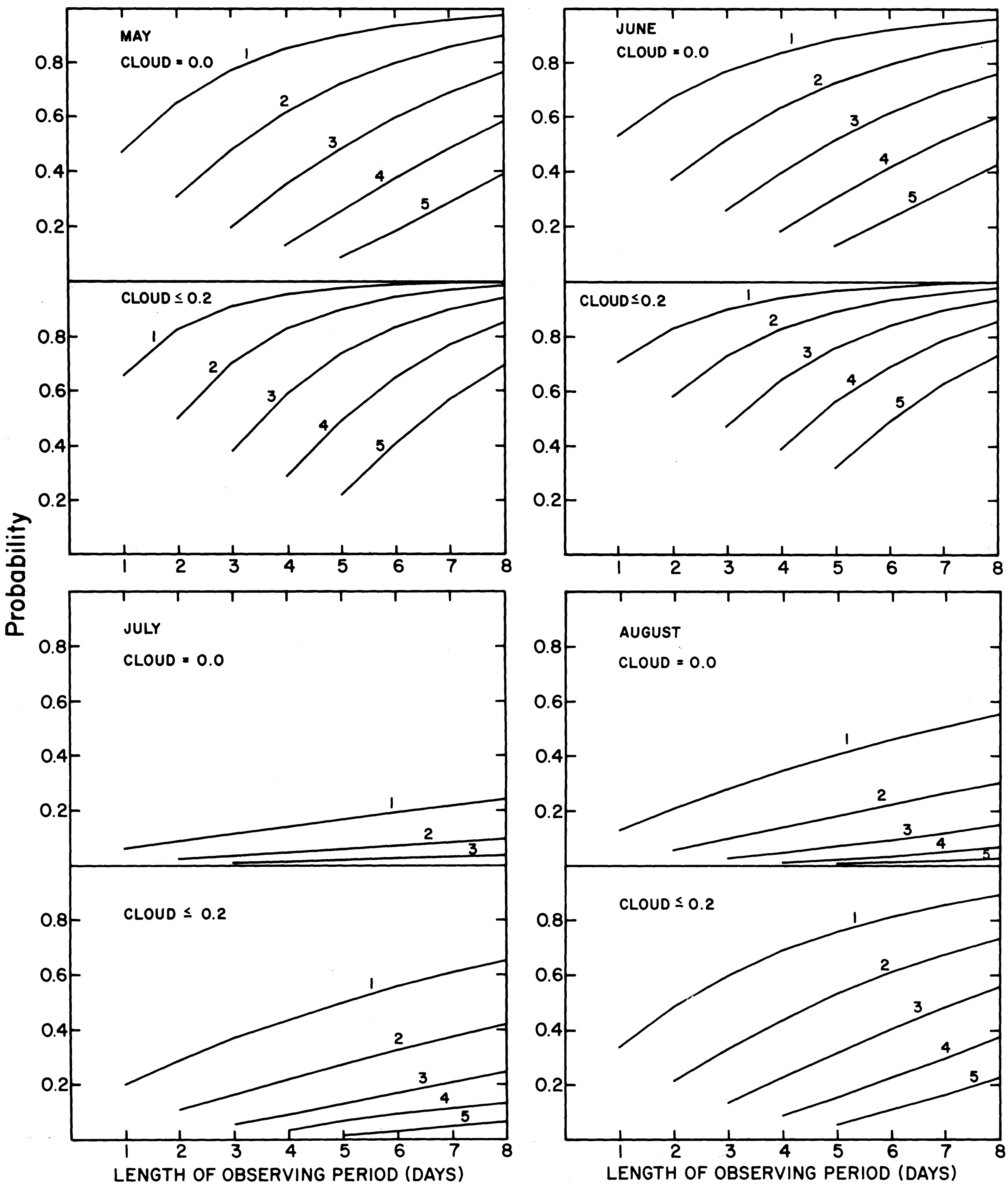

FIG. 2-2—The probability of obtaining not less than 1, 2, 3, 4, or 5 sunny days (indicated alongside the plots) during an arbitrary observing period of given length commencing in a given month. The probabilities vary greatly from month to month, being highest in June and May and lowest in July and August.

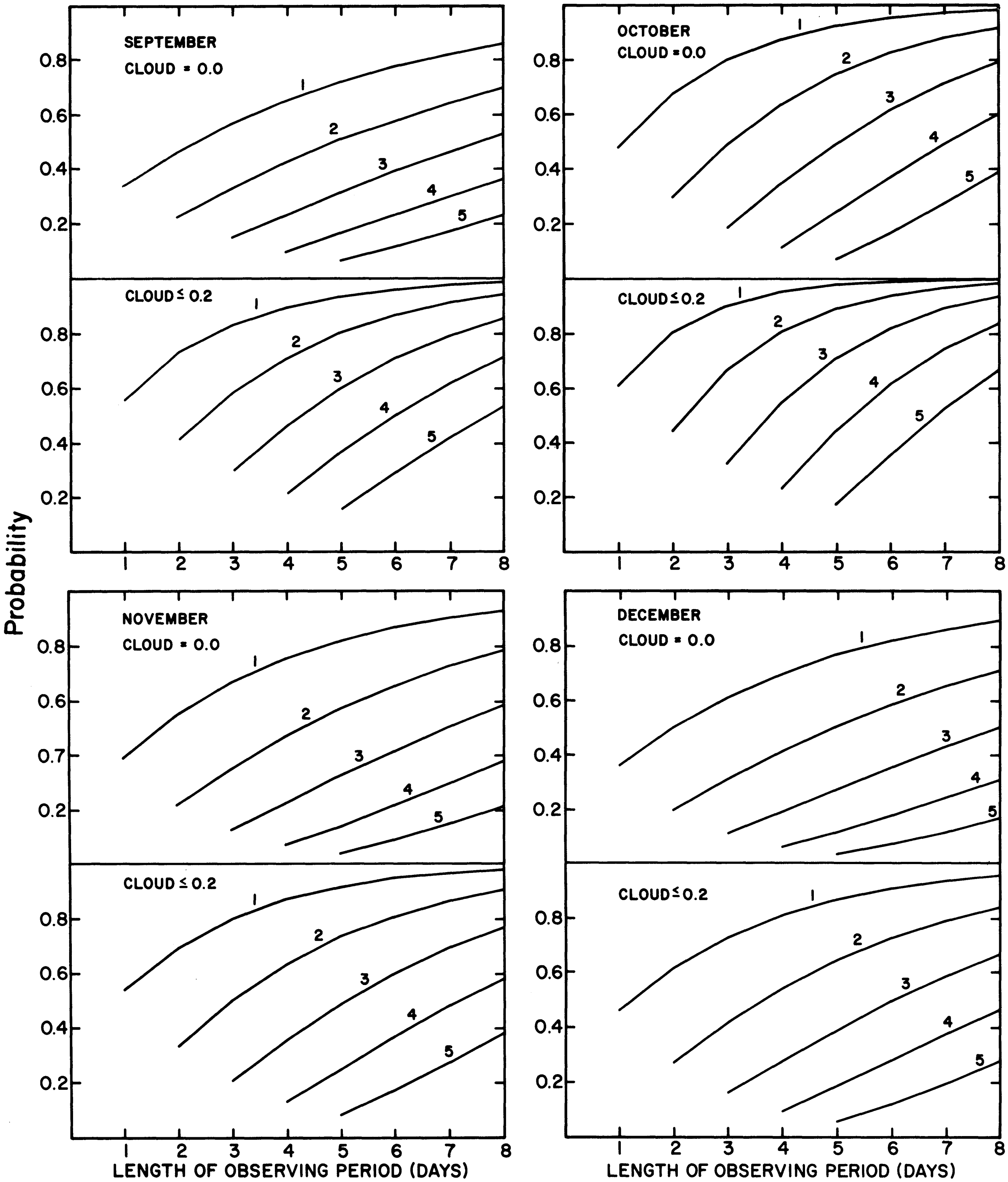

FIG. 2-3—The probability of obtaining not less than 1, 2, 3, 4, or 5 sunny days (indicated alongside the plots) during an arbitrary observing period of given length commencing in a given month. The probabilities vary greatly from month to month, being highest in June and May and lowest in July and August.